All Saints' CE Primary School

Key Stage 2 Calculation Policy

Year 3 National Curriculum objectives linked to addition and subtraction

These objectives are explicitly covered through the strategies outlined in this document:

- add and subtract numbers mentally, including: o a three-digit number and ones o a three-digit number and tens o a three-digit number and hundreds
- add and subtract numbers with up to four digits, using formal written methods of columnar addition and subtraction (four digits is Year 4)
- find 10 or 100 more or less than a given number
- find 1 000 more or less than a given number (Year 4)
- estimate the answer to a calculation and use inverse operations to check answers

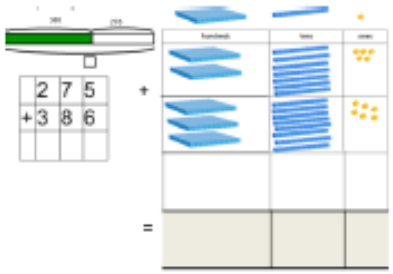
The following objectives should be planned for lessons where new strategies are being introduced and developed:

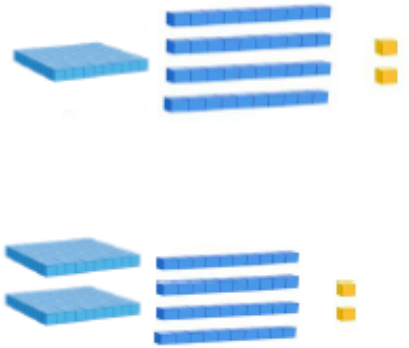
- solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

Year 3 Addition and Subtraction

Strategy & guidance	CPA
Add and subtract numbers mentally, including: • **a three-digit number and ones;** • **a three-digit number and tens;** • **a three-digit number and hundreds** *Pupils learn that this is an appropriate strategy when they are able to use known and derived number facts or other mental strategies to complete mental calculations with accuracy.* *To begin with, some pupils will prefer to use this strategy only when there is no need to regroup, using number facts within 10 and derivations. More confident pupils might choose from a range of mental strategies that avoid written algorithms, including (but not exhaustively):* • *known number facts within 20,* • *derived number facts,* • *'Make ten',* *See Year 2 guidance for exemplification of these – the use of concrete manipulatives other than Dienes blocks is important in reinforcing the use of these strategies.* *It is important that pupils are given plenty of (scaffolded) practice at choosing their own strategies to complete calculations efficiently and accurately. Explicit links need to be made between familiar number facts and the calculations that they can be useful for and pupils need to be encouraged to aim for efficiency.*	It is important to model the mental strategy using concrete manipulatives in the first instance and pupils should be able to exemplify their own strategies using manipulatives if required, with numbers appropriate to the unit they are working on (3-digit numbers in Units 1 & 4; 4-digit numbers in Unit 13). However, pupils should be encouraged to use known facts to derive answers, rather than relying on counting manipulatives or images. <u>No regrouping</u> 345 + 30 274 - 50 1128 + 300 1312 - 300 326 + 342 856 - 724  I know 4 + 3 = 7, so 4 tens plus 3 tens is equal to 7 tens. 345 + 30 = 375. <u>With some regrouping</u> 416 + 25 232 - 5 383 + 130 455 - 216 611 + 194 130 - 40 1482 + 900 2382 - 500

Year 3 Addition and Subtraction

Strategy & guidance	CPA
Written column method for calculations that require regrouping with up to 4-digits *Dienes blocks should be used alongside the pictorial representations during direct teaching and can be used by pupils both for support and challenge. Place value counters can also be introduced at this stage.* *This work revises and reinforces ideas from Key Stage 1, including the focus on place value – see Year 2 exemplification.* *Direct teaching of the columnar method should require at least one element of regrouping, so that pupils are clear about when it is most useful to use it. Asking them 'Can you think of a more efficient method?' will challenge them to apply their number sense / number facts to use efficient mental methods where possible.* *As in Year 2, pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping. In Year 3 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.* *Pupils should be challenged as to whether this is the most efficient method, considering whether mental methods (such as counting on, using known number facts, round and adjust etc.) may be likelier to produce an accurate solution.* *Pupils requiring support might develop their confidence in the written method using numbers that require no regrouping.*	As for the mental strategies, pupils should be exposed to concrete manipulatives modelling the written calculations and should be able to represent their written work pictorially or with concrete manipulatives when required. Again, they should be encouraged to calculate with known and derived facts and should not rely on counting images or manipulatives.  5 + 6 = 11 so I will have 11 ones which I regroup for 1 ten and 1 one. <u>Regrouping (including multiple separate instances)</u> 672 + 136 734 – 82 468 + 67 831 - 76 275 + 386 435 – 188 <u>'Regrouping to regroup'</u> 204 – 137 1035 - 851

Strategy & guidance	CPA
Find 10, 100 more or less than a given number *As pupils become familiar with numbers up to 1000, place value should be emphasised and comparisons drawn between adding tens, hundreds (and, in the last unit of the Summer term, thousands), including use of concrete manipulatives and appropriate images.* *After initial teaching, this should be incorporated into transition activities and practised regularly.*	142 + 100 = 242 

Column addition method in more detail

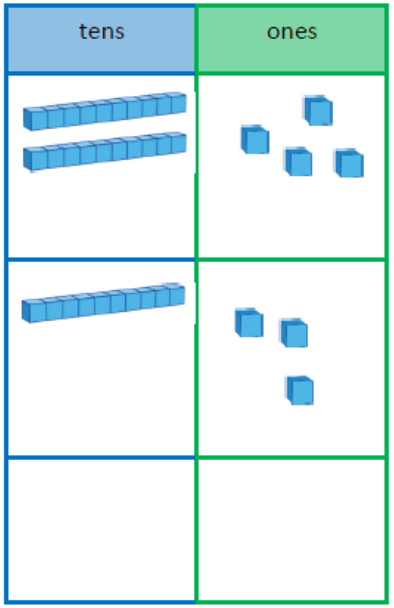
When pupils begin to solve addition of two 2-digit numbers in Mathematics Mastery they are introduced to the column addition method as a way of laying out the addition in columns that represent place value. This is first introduced in Year one and will continue to be used throughout pupils' primary education.

Column addition is a method that builds on pupils understanding of place value and different strategies including knowledge of number bonds within 20 and the 'make ten strategy'. One key misconception pupils may have when solving column addition and subtraction is to consider each digit as separate numbers rather than as representation of the number of tens or ones.
Below is a sequence for teaching how to solve addition using the column addition method, firstly without regrouping and secondly with regrouping.

<u>Column addition without regrouping</u>

24 + 13 =

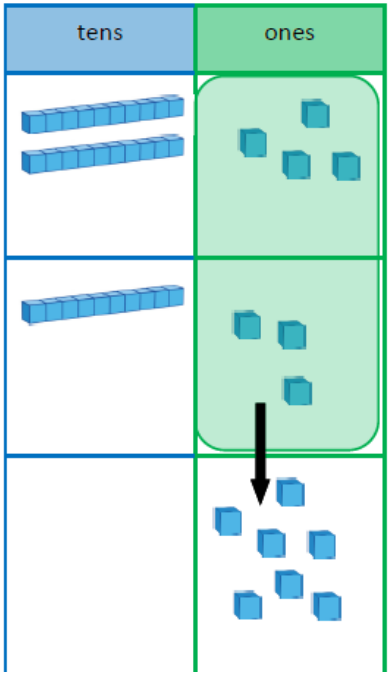
	tens	ones
	2	4
+	1	3



1. First add the ones

	tens	ones
	2	4
+	1	3
		7

4 ones + 3 ones = 7 ones

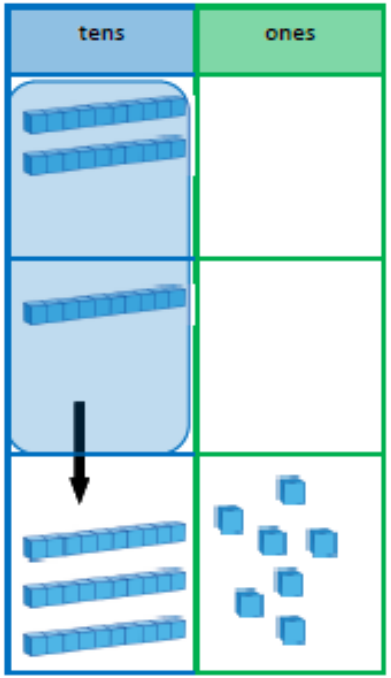


2. Then add the tens

	tens	ones
	2	4
+	1	3
	3	7

2 tens + 1 ten = 3 tens

So, 24 + 13 = 37

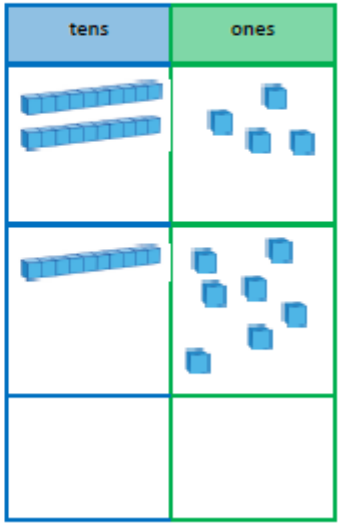


This written method is a very abstract representation of the equation and therefore teachers must make clear links between the written record and using manipulatives that reinforce place-value such as Dienes blocks. This must be planned for when teaching addition both when regrouping is and isn't required.

Column addition with regrouping

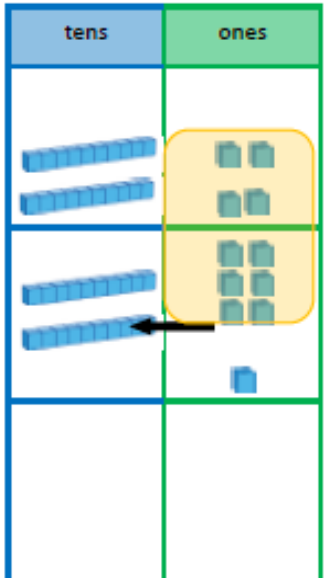
24 + 17 =

	tens	ones
	2	4
+	1	7



1. First add the ones

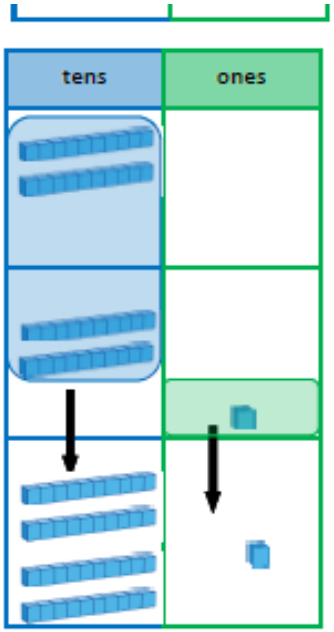
	tens	ones
	2	4
+	1	7
	1	
		1



4 ones + 7 ones = 11 ones

Regroup the ones.

11 ones = 1 ten and 1 one

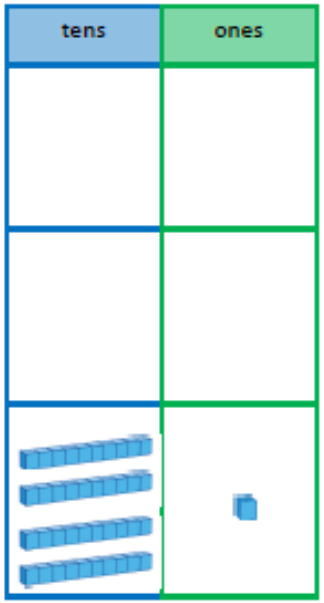


2. Then add the tens.

	tens	ones
	2	4
+	1	7
	1	
	4	1

2 tens + 1 ten + 1 ten = 4 tens

So, 24 + 17 = 41



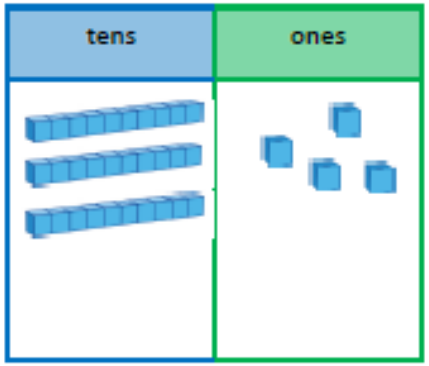
Column subtraction method in more detail

When pupils begin to solve subtraction of two 2-digit numbers in Mathematics Mastery they are introduced to the column subtraction method as a way of laying out the subtraction in columns that represent place value. This is first introduced in Year one and will continue to be used throughout pupils' primary education.

Column subtraction is a method that builds on pupils understanding of place value and different strategies including knowledge of number bonds within 20 and the 'make ten strategy'. One key misconception pupils may have when solving column addition and subtraction is to consider each digit as separate numbers rather than as representation of the number of tens or ones.
Below is a sequence for teaching how to solve subtraction using the column subtraction method, firstly without regrouping and secondly with regrouping.

Column subtraction without regrouping

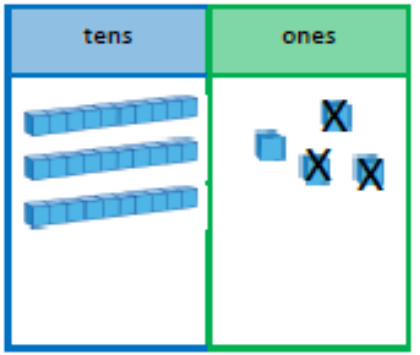
	tens	ones
	3	4
-	1	3



1. First subtract the ones

	tens	ones
	3	4
-	1	3
		1

4 ones - 3 ones = 1 one

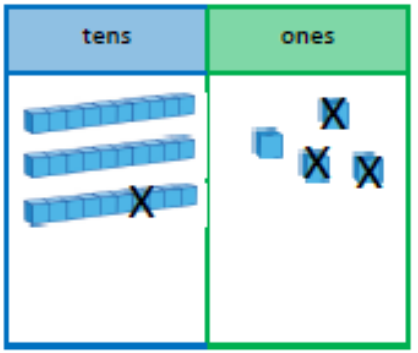


2. Then subtract the tens

	tens	ones
	3	4
-	1	3
	2	1

3 tens - 1 ten = 2 tens

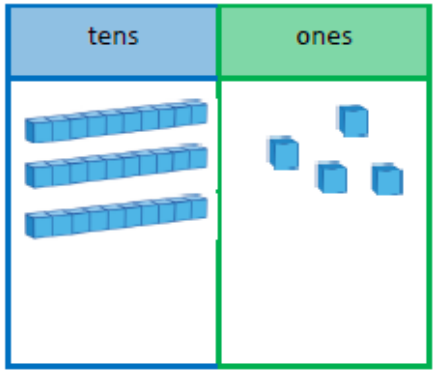
So, 34 - 13 = 21



This written method is a very abstract representation of the equation and therefore teachers must make clear links between the written record and using manipulatives that reinforce place-value such as Dienes blocks. This must be planned for when teaching subtraction both when regrouping is and isn't required.

Column subtraction with regrouping

	tens	ones
	3	4
-	1	7



1. First subtract the ones

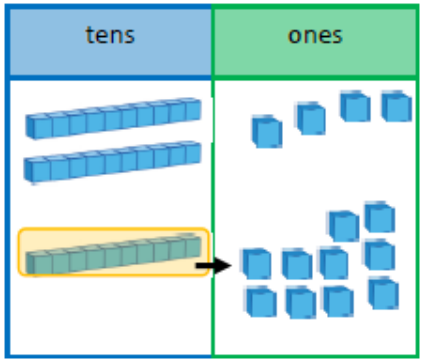
But we cannot subtract 7 ones from 4 ones.

So, we regroup the tens in 34.

Regroup the tens in 34.

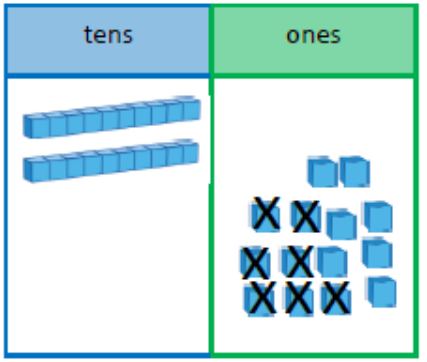
34 = 3 tens and 4 ones

34 = 2 tens and 14 ones



First subtract the ones

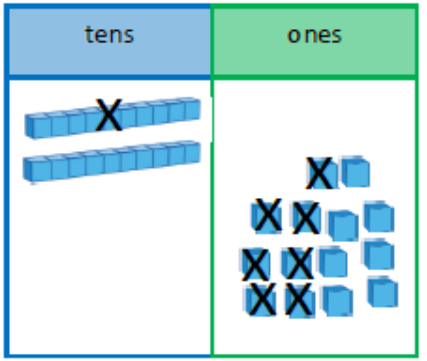
	tens	ones
	$^{2}\not{3}$	$^{1}4$
-	1	7
		7



14 ones - 7 ones = 7 ones

2. Then subtract the tens

	tens	ones
	$^{2}\not{3}$	$^{1}4$
-	1	7
	1	7



2 tens - 1 ten = 1 tens

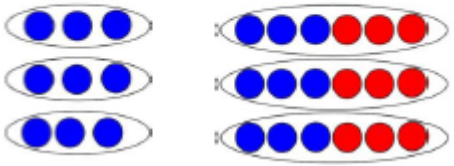
So, 34 - 17 = 17

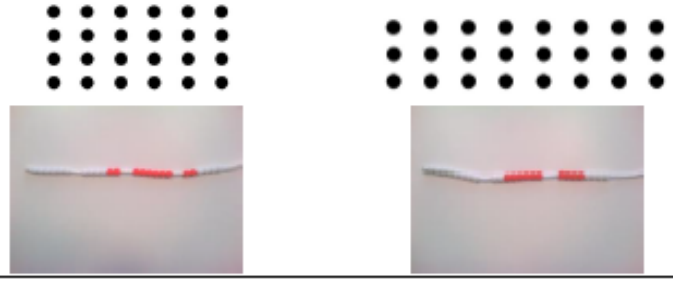
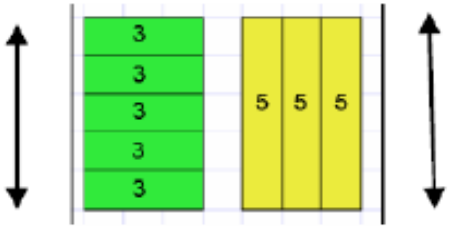
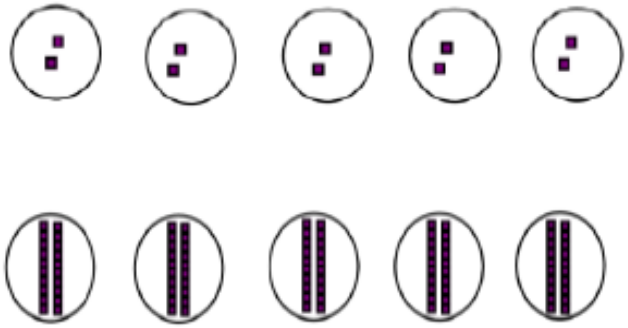
Year 3 National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

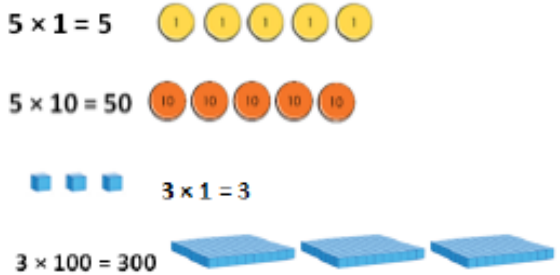
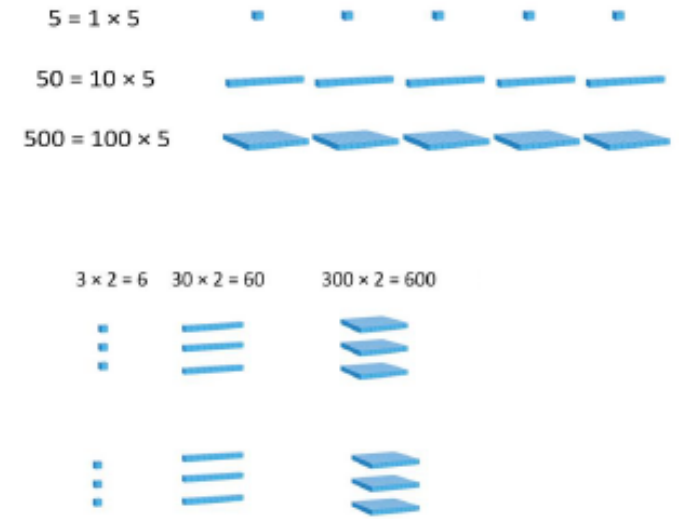
- count from 0 in multiples of 4, 8, 50 and 100
- recall and use multiplication and division facts for the 3, 4, 6, and 8 multiplication tables
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental methods
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which *n* objects are connected to *m* objects

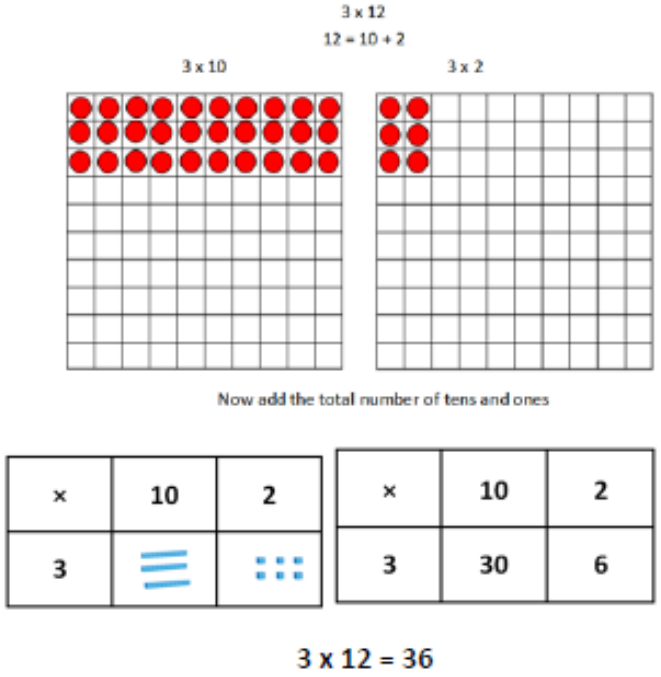
Year 3 Multiplication

Strategy & guidance	CPA
Doubling to derive new multiplication facts *Pupils continue to make use of the idea that facts from easier times tables can be used to derive facts from related times tables using doubling as a strategy.* *This builds on the doubling strategy from Year 2.*	3 x 3 = 9 3 x 6 = double 9 = 18 

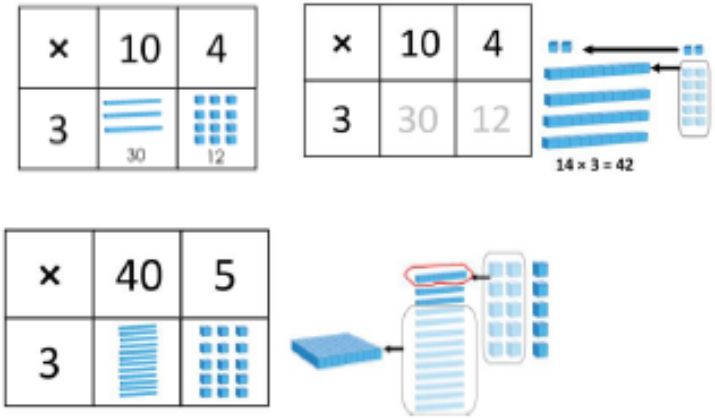
Strategy & guidance	CPA
Skip counting in multiples of 2, 3, 4, 5, 6, 8 and 10 *Rehearsal of previously learnt tables as well as new content for Year 3 should be incorporated into transition activities and practised regularly.*	
Use of part-part-whole model with arrays and bar models to establish commutativity and inverse relationship between multiplication and division *In these contexts pupils are able to identify all the equations in a fact family.*	 
Ten times greater *Pupils's work on this must be firmly based on concrete representations – the language of ten times greater must be well modelled and understood to prevent the numerical misconception of 'adding a zero'.*	

Year 3 Multiplication

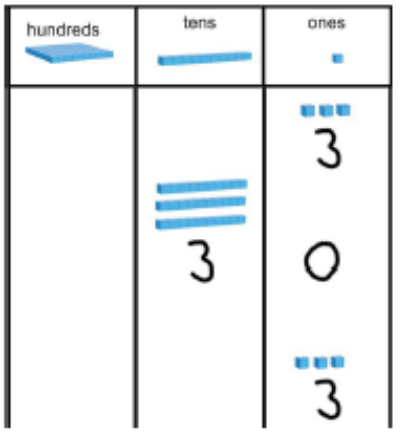
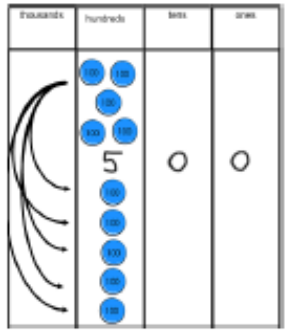
Strategy & guidance	CPA
Multiplying by 10 and 100 *Building on the ten times greater work, pupils use appropriate Dienes blocks and place value counters to multiply 2, 3, 4, 5 and 10 by 10, 100 and 1000.*	
Using known facts for multiplying by multiples of 10 and 100 *Pupils' growing understanding of place value, allows them to make use of known facts to derive multiplications using powers of 10.* *It is important to use tables with which they are already familiar (i.e. not 7 or 9 tables in Year 3)*	

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (no regrouping) *Children should always consider whether partitioning is the best strategy – if it is possible to use strategies such as doubling (some may use doubling twice for ×4), they need to choose the most efficient strategy.* *Children may wish to make jottings, including a full grid as exemplified here – but grid method is not a formal method and its only purpose is to record mental calculations. This supports the development of the necessary mental calculating skills but does not hinder the introduction of formal written methods in Year 4. Concrete manipulatives are essential to develop understanding.*	

Year 3 Multiplication

Strategy & guidance	CPA
Multiplication of 2-digit numbers with partitioning (regrouping) *Using concrete manipulatives and later moving to using images that represent them, supports pupils' early understanding, leading towards formal written methods in Year 4.* *Once again, this is a mental strategy, which they may choose to support with informal jottings, including a full grid, as exemplified here.* *Pupils must be encouraged to make use of their known multiplication facts and their knowledge of place value to calculate, rather than counting manipulatives.*	

Year 3 Division

Strategy & Guidance	CPA
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using scaling down *Pupils use the strategy of 'scaling down', representing numbers with concrete manipulatives and making the value ten times smaller.*	 3 × 10 = 30 30 ÷ 10 = 3
Dividing multiples of 10, 100 and 1000 by 10, 100 and 1000 using grouping *Pupils divide by 10, 100 and 1000 by making groups of the divisor.*	500 ÷ 100 = ☐ My whole is 500 and the value of the equal parts is 100. How many parts are there? 

Year 4 National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

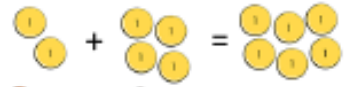
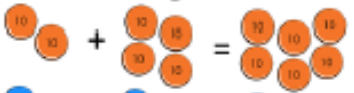
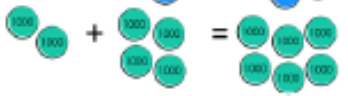
- add and subtract numbers with up to four digits, using the formal written methods of columnar addition and subtraction where appropriate
- find 1 000 more or less than a given number
- estimate and use inverse operations to check answers to a calculation

N.B. There is no explicit reference to mental calculation strategies in the programmes of study for Year 4 in the national curriculum. However, with an overall aim for fluency, appropriate mental strategies should always be considered before resorting to formal written procedures, with the emphasis on pupils making their own choices from an increasingly sophisticated range of strategies.

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why
- solve simple measure and money problems involving fractions and decimals to two decimal places

Year 4 Addition and Subtraction

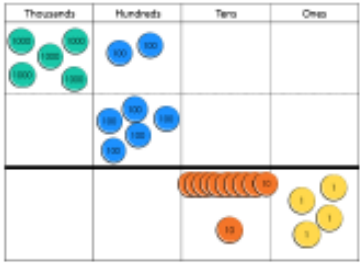
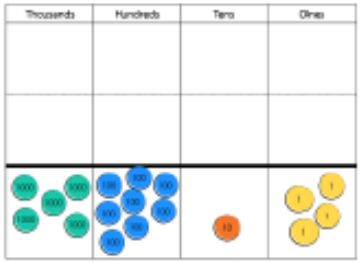
Strategies & Guidance	CPA
Count forwards and backwards in steps of 10, 100 and 1000 for any number up to 10 000. *Pupils should count on and back in steps of ten, one hundred and one thousand from different starting points. These should be practised regularly, ensuring that boundaries where more than one digit changes are included.* **Count forwards and backwards in tenths and hundredths**	  Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. E.g. 990 + 10 or 19.9 + 0.1
Using known facts and knowledge of place value to derive facts. **Add and subtract multiples of 10, 100 and 1000 mentally** *Pupils extend this knowledge to mentally adding and subtracting multiples of 10, 100 and 1000. Counting in different multiples of 10, 100 and 1000 should be incorporated into transition activities and practised regularly.*	 2 + 4 = 6  20 + 40 = 60  200 + 400 = 600  2000 + 4000 = 6000
Adding and subtracting by partitioning one number and applying known facts. *By Year 4 pupils are confident in their place value knowledge and are calculating mentally both with calculations that do not require regrouping and with those that do.*	See Y3 guidance on mental addition & subtraction, remembering that use of concrete manipulatives and images in both teaching and reasoning activities will help to secure understanding and develop mastery.

Year 4 Addition and Subtraction

Near doubles

Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten. These facts can be adjusted to calculate near doubles.

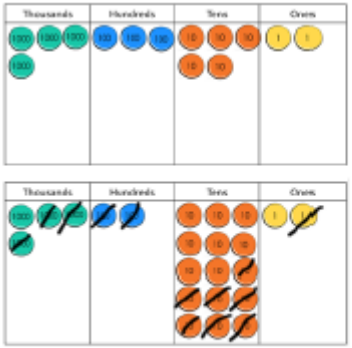
1600 + 1598 = double 1600 – 2

Strategies & Guidance	CPA
Written column methods for addition *Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.*	  

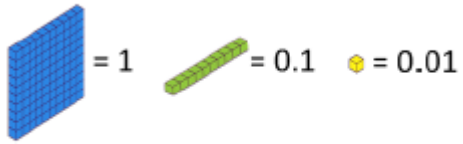
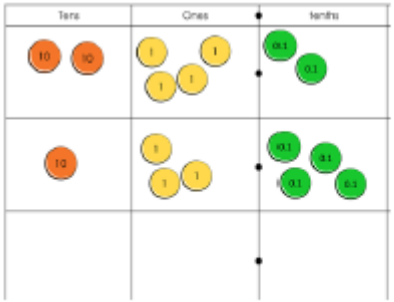
	5	2	7	3
+		5	4	1
	5	8	1	4

Written column methods for subtraction

Place value counters are a useful manipulative for representing the steps of the formal written method. These should be used alongside the written layout to ensure conceptual understanding and as a tool for explaining.



$$\begin{array}{r} 4\,^{2}\not{3}\,^{1}5\,2 \\ -\ 3\,2\,7\,1 \\ \hline 1\,0\,8\,1 \end{array}$$

Strategies & Guidance	CPA
Calculating with decimal numbers *Assign different values to Dienes equipment. If a Dienes 100 block has the value of 1, then a tens rod has a value of 0.1 and a ones cube has a value of 0.01. These can then be used to build a conceptual understanding of the relationship between these.* *Place value counters are another useful manipulative for representing decimal numbers.* *All of the calculation strategies for integers (whole numbers) can be used to calculate with decimal numbers.*	 24.2 + 13.4 = 

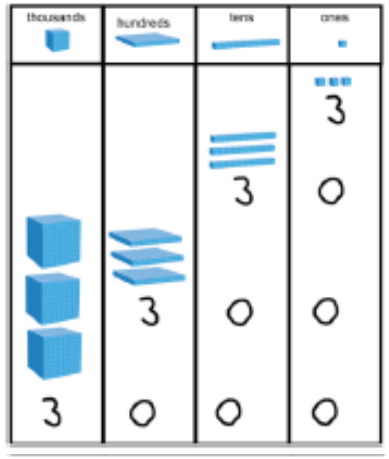
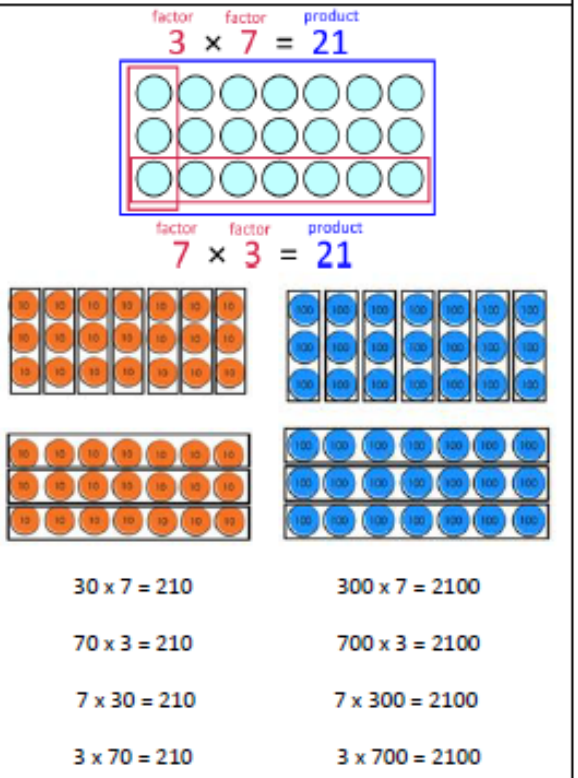
National Curriculum objectives linked to multiplication and division

These objectives are explicitly covered through the strategies outlined in this document:

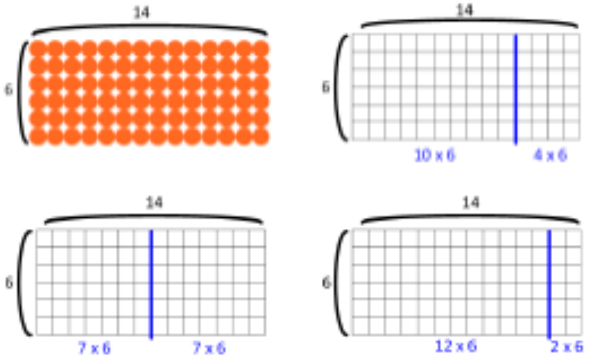
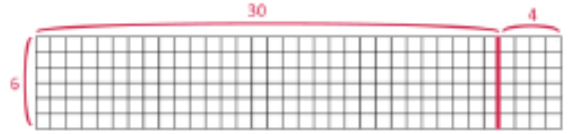
- count from 0 in multiples of 6, 7, 9, 25 and 1000
- recall and use multiplication and division facts for multiplication tables up to 12 × 12
- write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods
- recognise and use factor pairs and commutativity in mental calculations
- use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths.

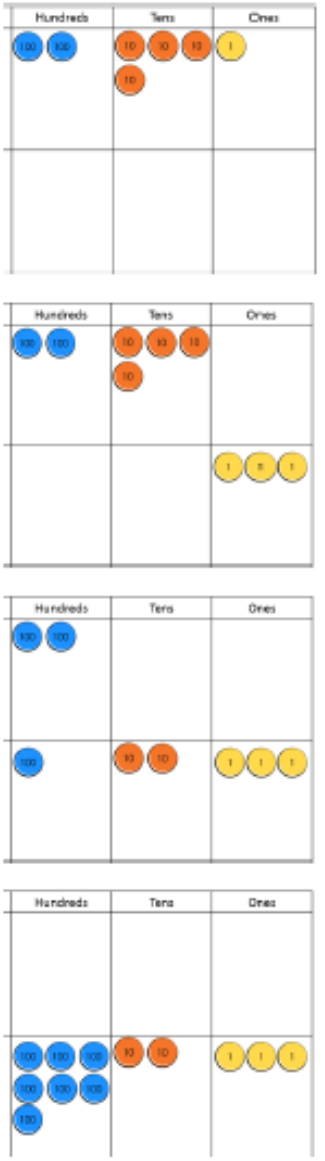
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as *n* objects are connected to *m* objects.

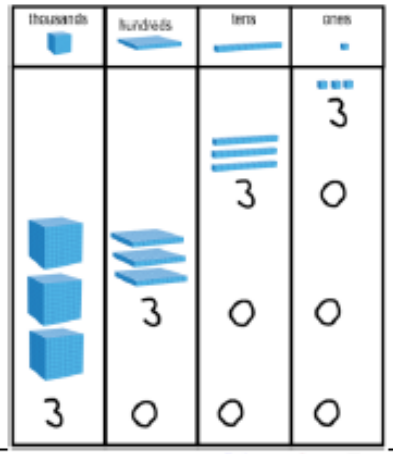
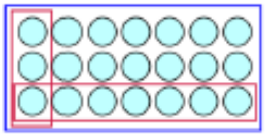
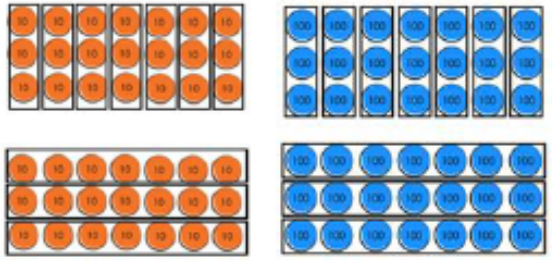
Strategies & Guidance	CPA
Multiplying by 10 and 100 *When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc.* *When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater.* *Repeated multiplication by ten will build an understanding of multiplying by 100 and 1000*	
Using known facts and place value for mental multiplication involving multiples of 10 and 100 *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity (that multiplication can be completed in any order) is used to find a range of related facts.*	

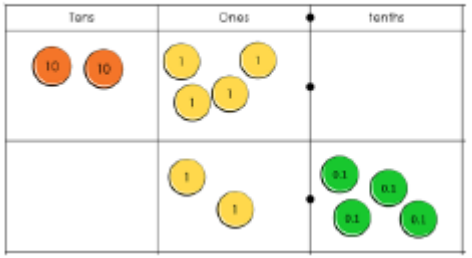
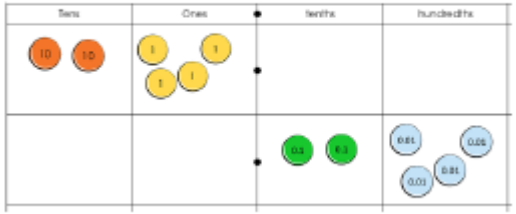
Year 4 Multiplication

Strategies & Guidance	CPA
Multiplying by partitioning one number and multiplying each part *Pupils build on mental multiplication strategies and develop an explicit understanding of distributive law, which allows them to explore new strategies to make more efficient calculations.* *As well as partitioning into tens and ones (a familiar strategy), they begin to explore compensating strategies and factorisation to find the most efficient solution to a calculation.* **Distributive law** **a x (b + c) = a x b + a x c**	14 x 6  34 × 6  30 × 6 + 4 × 6 
Mental multiplication of three 1-digit numbers, using the associative law *Pupils first learn that multiplication can be performed in any order, before applying this to choose the most efficient order to complete calculations, based on their increasingly sophisticated number facts and place value knowledge.*	Four pots each containing two flowers which each have seven petals. How many petals in total?  (4 x 2) x 7 or 4 x (2 x 7)

Strategies & Guidance	CPA
Short multiplication of 3-digit number by 1-digit number *To begin with pupils are presented with calculations that require no regrouping or only regrouping from the ones to the tens. Their conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.* *With practice pupils will be able to regroup in any column, including from the hundreds to the thousands, including being able to multiply numbers containing zero and regrouping through multiple columns in a single calculation.*	Exemplification of this process is best understood through viewing the video tutorial  To calculate 241 x 3, represent the number 241. Multiply each part by 3, regrouping as needed. 2 4 1 x 3 7 2 3 1

Year 4 Division

Strategies & Guidance	CPA
Dividing by 10 and 100 *When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller.* *When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21*	 30 ÷ 10 = 3 300 ÷ 100 = 3 3000 ÷ 1000 = 3 300 ÷ 10 = 30 3000 ÷ 100 = 30 3000 ÷ 10 = 300
Derived facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	21 ÷ 3 = 7  21 ÷ 7 = 3  210 ÷ 7 = 30 210 ÷ 3 = 70 210 ÷ 30 = 7 210 ÷ 70 = 3 2100 ÷ 7 = 300 2100 ÷ 3 = 700 2100 ÷ 300 = 7 2100 ÷ 700 = 3

Strategies & Guidance	CPA
Short division of 4-digit numbers by 1-digit numbers *Pupils start with dividing 4-digit numbers by 2, 3 and 4, where no regrouping is required. Place value counters are used simultaneously in a place value chart, to develop conceptual understanding.* *They progress to calculations that require regrouping in the hundreds or tens columns.* *Pupils build on their conceptual knowledge of division to become confident with dividing numbers where the tens digit is smaller than the divisor, extending this to any digit being smaller than the divisor.*	**Division as sharing**  **Division as grouping** 
Division of a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths *When you divide by ten, each part is ten times smaller. The tens become ones and the ones become tenths. Each digit is in a place that gives it a value that is ten times smaller.*	24 ÷ 10 = 2.4  24 ÷ 100 = 0.24 

Year 5 and 6 National Curriculum objectives linked to integer addition and subtraction

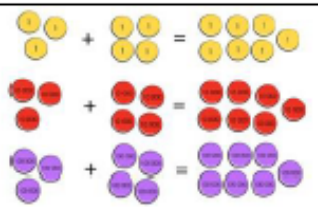
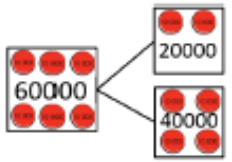
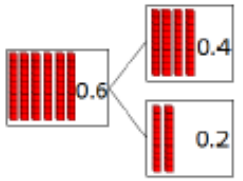
Year 5 and Year 6 are together because the calculation strategies used are broadly similar, with Year 6 using larger and smaller numbers. Any differences for Year 6 are highlighted in red.

These objectives are explicitly covered through the strategies outlined in this document:

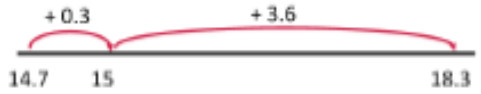
- add and subtract numbers mentally with increasingly large numbers
- add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)
- use negative numbers in context, and calculate intervals across zero
- perform mental calculations, including with mixed operations and large numbers
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

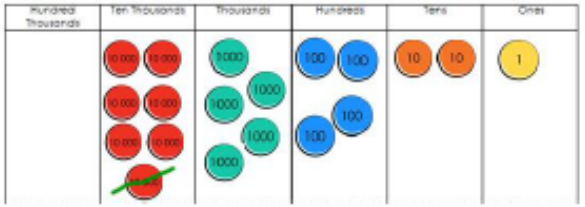
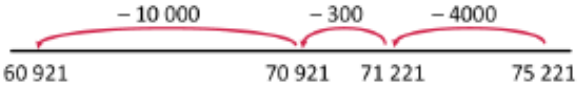
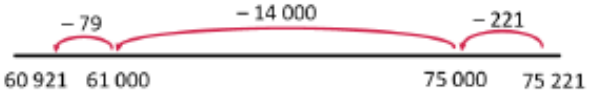
The following objectives should be planned for lessons where new strategies are being introduced and developed:

- use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign.

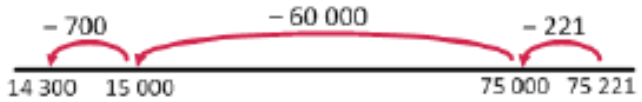
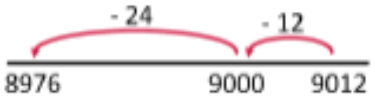
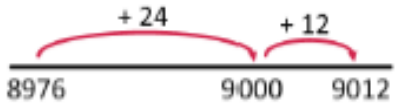
Strategies & Guidance	CPA
Count forwards or backwards in steps of powers of 10 for any given number up to 1 000 000 *Skip counting forwards and backwards in steps of powers of 10 (i.e. 10, 100, 1000, 10 000 and 100 000) should be incorporated into transition activities and practised regularly.* *In Year 5 pupils work with numbers up to 1 000 000 as well as tenths, hundredths and thousandths.* *In Year 6 pupils work with numbers up to 10 000 000.*	Support with place value counters on a place value chart, repeatedly adding the same counter and regrouping as needed.  Counting sticks and number lines:   Pay particular attention to boundaries where regrouping happens more than once and so more than one digit changes. e.g. 9900 + 100 = 10 000 or 99 000 + 1000 = 100 000
Using known facts and understanding of place value to derive *Using the following language makes the logic explicit: I know three ones plus four ones is equal to seven ones. Therefore, three ten thousands plus four ten thousands is equal to seven ten thousands.* *In Year 5 extend to multiples of 10 000 and 100 000 as well as tenths, hundredths and thousandths.* *In Year 6 extend to multiples of one million.* *These derived facts should be used to estimate and check answers to calculations.*	 3 + 4 = 7 30 000 + 40 000 = 70 000 300 000 + 400 000 = 700 000 20 000 + 40 000 = 60 000 40 000 + 20 000 = 60 000 60 000 - 40 000 = 20 000 60 000 - 20 000 = 40 000   0.6 = 0.2 + 0.4 0.6 = 0.4 + 0.2 0.2 = 0.6 - 0.4 0.4 = 0.6 - 0.2

Year 5 and 6 Addition and Subtraction

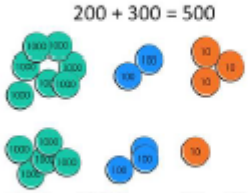
Strategies & Guidance	CPA
Partitioning one number and applying known facts to add. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be added.* *They should be encouraged to count on from the number of greater value as this will be more efficient. However, they should have an understanding of the commutative law of addition, that the parts can be added in any order.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 4650 + 7326 = 7326 + 4000 + 600 + 50  With place value counters, represent the larger number and then add each place value part of the other number. The image above shows the thousands being added. Represent pictorially with an empty numberline:  **Partitioning in different ways (non-canonical partitioning):** Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count on to a multiple of 10. 6785 + 2325 = 6785 + 15 + 200 + 2110  The strategy can be used with decimal numbers, Make one: 14.7 + 3.6 = 14.7 + 0.3 + 3.3 = 15 + 3.3 

Strategies & Guidance	CPA
Subtraction by partitioning and applying known facts. *Pupils can use this strategy mentally or with jottings as needed.* *Pupils should be aware of the range of choices available when deciding how to partition the number that is to be subtracted.* *Pupils have experience with these strategies with smaller numbers from previous years and so the focus should be on developing flexibility and exploring efficiency.*	**Partitioning into place value amounts (canonical partitioning):** 75 221 – 14 300 = 75 221 – 10 000 – 4000 – 300  Represent pictorially with a number line, starting on the right and having the arrows jump to the left:  Develop understanding that the parts can be subtracted in any order and the result will be the same:  **Partitioning in different ways (non-canonical partitioning):** Extend the 'Make ten' strategy (see guidance in Y1 or Y2) to count back to a multiple of 10. 

Year 5 and 6 Addition and Subtraction

Strategies & Guidance	CPA
Calculate difference by "counting back" *It is interesting to note that finding the difference is reversible. For example, the difference between 5 and 2 is the same as the difference between 2 and 5. This is not the case for other subtraction concepts.*	75 221 – 14 300 Place the numbers either end of a numberline and work out the difference between them. Select efficient jumps.  Finding the difference is efficient when the numbers are close to each other: 9012 – 8976 
Calculate difference by "counting on" *Addition strategies can be used to find difference.*	75 221 – 14 300  Finding the difference is efficient when the numbers are close to each other 9012 – 8976 

Near doubles *Pupils should be able to double numbers up to 100 and use this to derive doubles for multiples of ten as well as decimal numbers. These facts can be adjusted to calculate near doubles.*	160 + 170 = double 150 + 10 + 20 160 + 170 = double 160 + 10 or 160 + 170 = double 170 - 10 2.5 + 2.6 = double 2.5 + 0.1

Strategies & Guidance	CPA
Partition both numbers and combine the parts *Pupils should be secure with this method for numbers up to 10 000, using place value counters or Dienes to show conceptual understanding.* *If multiple regroupings are required, then pupils should consider using the column method.*	7230 + 5310 = 12 000 + 500 + 40 200 + 300 = 500  7000 + 5000 = 12000 30 + 10 = 40 Pupils should be aware that the parts can be added in any order.

Year 5 and 6 Addition and Subtraction

Written column methods for addition

In Year 5, pupils are expected to be able to use formal written methods to add whole numbers with more than four digits as well as working with numbers with up to three decimal places.

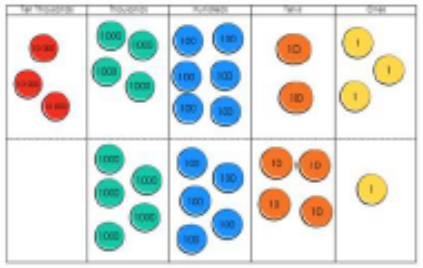
Pupils should think about whether this is the most efficient method, considering if mental methods would be more effective.

Continue to use concrete manipulatives alongside the formal method.

When adding decimal numbers with a different number of decimal places, in order to avoid calculation errors, pupils should be encouraged to insert zeros so that there is a digit in every row. This is not necessary for calculation and these zeros are not place holders as the value of the other digits is not changed by it being placed.

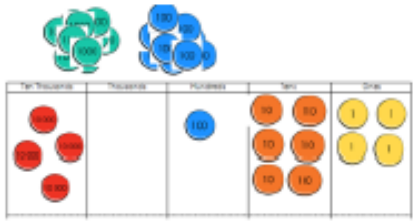
For this method start with the digit of least value because if regrouping happens it will affect the digits of greater value.

```
  3 4 6 2 3
+   5 5 4 1
-----------
```



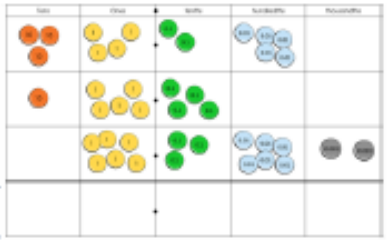
Combine the counters in each column and regroup as needed:

```
  3 4 6 2 3
+   5 5 4 1
  1   1
-----------
  4 0 1 6 4
```



Decimal numbers:

```
  3 4 . 2 5
  1 5 . 4
+   6 . 3 6 2
-------------
  5 6 . 0 1 2
```



Strategies & Guidance	CPA

Written column methods for subtraction

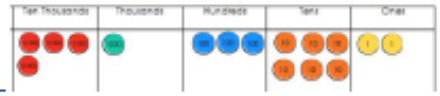
In Year 5, pupils are expected to be able to use formal written methods to subtract whole numbers with more than four digits as well as working with numbers with up to three decimal places.

Pupils should be given plenty of practice with calculations that require multiple separate instances of regrouping.

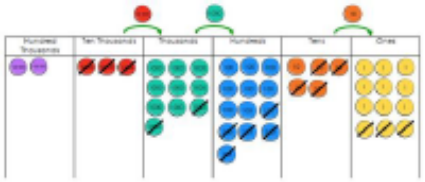
In Year 3 and 4 they become more familiar with calculations that require 'regrouping to regroup'. Understanding must be secured through the considered use of manipulatives and images, combined with careful use of language.

Pupils should think about if this is the most efficient method, considering whether mental strategies (such as counting on, using known number facts, compensation etc.) may be likelier to produce an accurate solution.

```
  4 1 3 6 2
- 3 2 2 4 3
-----------
```



```
  3 1 5 1 2
  4
- 3 2 2 4 3
-----------
  9 1 1 9
```



The term regrouping should be the language used. You can use the terms 'exchange' with subtraction but it needs careful consideration.

You can regroup 62 as 50 and 12 (5 tens and 12 ones) instead of 60 and 2 (6 tens and 12 ones).

Or you can 'exchange' one of the tens for 10 ones resulting in 5 tens and 12 ones.

If you have exchanged, then the number has been regrouped.

Year 5 and Year 6 National Curriculum objectives linked to multiplication and division

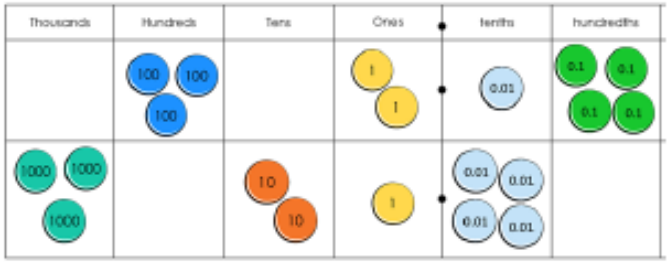
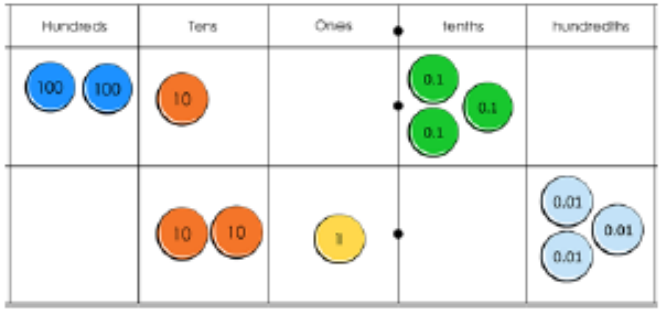
These objectives are explicitly covered through the strategies outlined in this document:

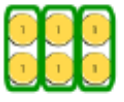
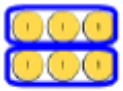
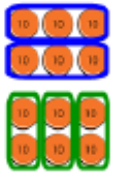
- multiply and divide whole numbers by 10, 100 and 1000
- multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers
- multiply and divide numbers mentally drawing upon known facts
- divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context
- multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication
- divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context
- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- multiply one-digit numbers with up to two decimal places by whole numbers
- use written division methods in cases where the answer has up to two decimal places

The following objectives should be planned for lessons where new strategies are being introduced and developed:

- solve problems involving multiplication and division including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- use their knowledge of the order of operations to carry out calculations involving the four operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- solve problems involving the relative sizes of two quantities where missing values can be found by using integer multiplication and division facts.

Year 5 and 6 Multiplication

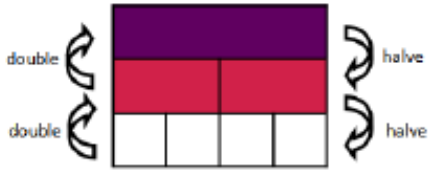
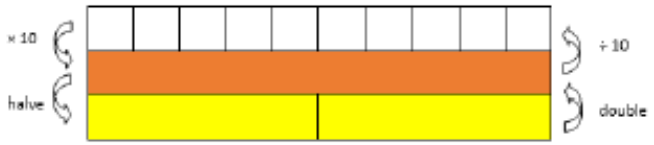
Strategies & Guidance	CPA
Multiply and divide whole numbers and those involving decimals by 10, 100 and 1000 *Avoid saying that you "add a zero" when multiplying by ten and instead use the language of place holder.* *Use place value counters and charts to visualise and then notice what happens to the digits.*	When you multiply by ten, each part is ten times greater. The ones become tens, the tens become hundreds, etc. When multiplying whole numbers, a zero holds a place so that each digit has a value that is ten times greater. 102.14 x 10 = 1021.4  When you divide by ten, each part is ten times smaller. The hundreds become tens and the tens become ones. Each digit is in a place that gives it a value that is ten times smaller. When dividing multiples of ten, a place holder is no longer needed so that each digit has a value that is ten times smaller. E.g. 210 ÷ 10 = 21 210.3 ÷ 10 = 21.03 

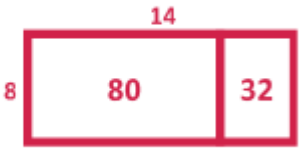
Strategies & Guidance	CPA
Using known facts and place value to derive multiplication facts *Emphasis is placed on understanding the relationship (10 times or 100 times greater) between a known number fact and one to be derived, allowing far larger 'fact families' to be derived from a single known number fact.* *Knowledge of commutativity is further extended and applied to find a range of related facts.* *Pupils should work with decimals with up to two decimal places.* *These derived facts should be used to estimate and check answers to calculations.*	  2 x 3 = 6 3 x 2 = 6 2 x 30 = 60 30 x 2 = 60   2 x 3 x 10= 60 3 x 20 = 60 20 x 3 = 60 2 x 300 = 600 300 x 2 = 600  3 x 200 = 600 200 x 3 = 600 2 x 30 x 10 = 600 20 x 3 x 10 = 600   20 x 30 = 600 30 x 20 = 600

These are the multiplication facts pupils should be able to derive from a known fact

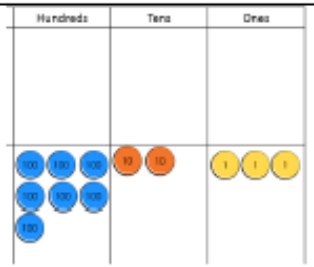
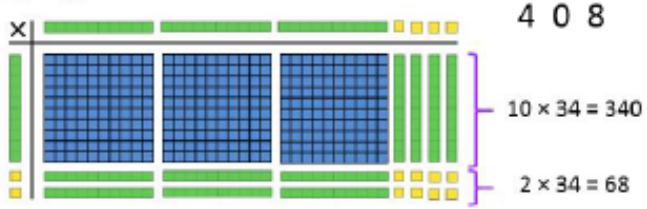
2 100 000		700 000 x 3	70 000 x 30	7000 x 300	700 x 3000	70 x 30 000	7 x 300 000
210 000		70 000 x 3	7000 x 30	700 x 300	70 x 3000	7 x 30 000	
21 000		7000 x 3	700 x 30	70 x 300	7 x 3000		
2100		700 x 3	70 x 30	7 x 300			
210		70 x 3	7 x 30				
21	=	**7 x 3**					
2.1		0.7 x 3	7 x 0.3				
0.21		0.07 x 3	0.7 x 0.3	7 x 0.03			
0.021		0.007 x 3	0.07 x 0.3	0.7 x 0.03	7 x 0.003		

Year 5 and 6 Multiplication

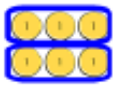
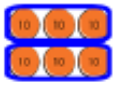
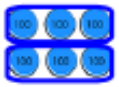
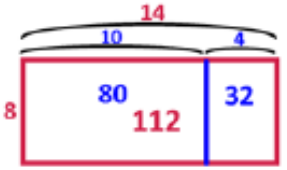
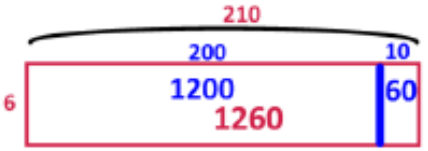
Strategies & Guidance	CPA
Doubling and halving *Pupils should experience doubling and halving larger and smaller numbers as they expand their understanding of the number system.* *Doubling and halving can then be used in larger calculations.*	 **Multiply by 4** by doubling and doubling again e.g. 16 × 4 = 32 × 2 = 64 **Divide by 4** by halving and halving again e.g. 104 ÷ 4 = 52 ÷ 2 = 26  **Multiply by 8** by doubling three times e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96 **Divide by 8** by halving three times e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13  **Multiply by 5** by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90. **Divide by 5** by dividing by 10 and doubling, e.g. 460 ÷ 5 = double 46 = 92

Strategies & Guidance	CPA
Multiply by partitioning one number and multiplying each part **Distributive law** **a x (b + c) = a x b + a x c** Build on pupils' understanding of arrays of counters to represent multiplication to see that area models can be a useful representation:	8 x 14 = 8 x 10 + 8 x 4  Cuisenaire rods to build arrays  Represent with area model  Jottings on a number line Bead string where each bead has a value of 8: 
Using knowledge of factors *In Year 5 pupils are expected to be able to identify factor pairs and this knowledge can be used to calculate.* *Pupils will be using the commutative and associative laws of multiplication.* **Commutative law** **a x b = b x a** **Associative law** **a x b x c = (a x b) x c** **= a x (b x c)** *They should explore and compare the different options and choose the most efficient order to complete calculations.*	Calculate 6 x 24 by using factor pairs of 24 Two and twelve are factors of 24: 6 x 2 x 12  6 x 12 x 2  Three and eight are factors of 24: 6 x 3 x 8  6 x 8 x 3  Four and six are factors of 24: 6 x 4 x 6  6 x 6 x 4 

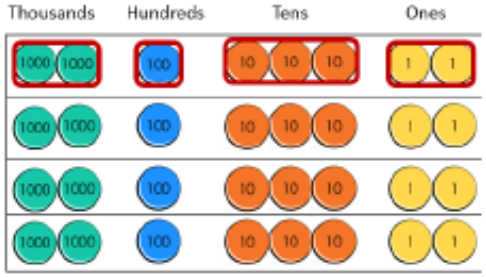
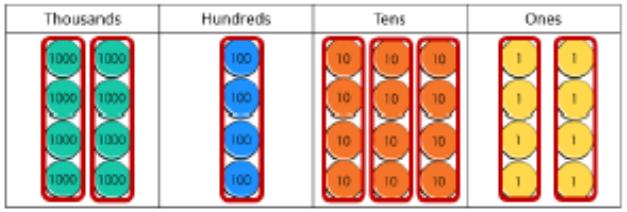
Year 5 and 6 Multiplication

Strategies & Guidance	CPA
Formal written method of short multiplication *Conceptual understanding is supported by the use of place value counters, both during teacher demonstrations and during their own practice.*	241 x 3 = 723 (1) 
Multiplying by a 2-digit number **Formal written method of long multiplication** *In Year 6 pupils are extended from multiplication by a 1-digit number to multiplication by a 2-digit number.* *Extend the place value chart model used in Year 4, using an additional row on the place value chart.* *Extend understanding of the distrubitive law to develop conceptual understanding of the two rows of the formal written method.* *Dienes blocks can be used to construct area models to represent this.*	243 x 12 = 486 + 2430  34 x 12 = 68 + 340 = 408 12 × 34 

Year 5 and 6 Division

Strategies & Guidance	CPA
Deriving facts from known facts *Pupils use their growing knowledge of multiplication facts, place value and derived facts to multiply mentally.* *Understanding of the inverse relationship between multiplication and division allows corresponding division facts to be derived.*	6 ÷ 2 = 3  6 ÷ 3 = 2 60 ÷ 2 = 30  60 ÷ 3 = 20 60 ÷ 30 = 2 60 ÷ 20 = 3 600 ÷ 2 = 300  600 ÷ 3 = 200 600 ÷ 300 = 2 600 ÷ 200 = 3
Using knowledge of multiples to divide *Using an area model to partition the whole into multiples of the divisor (the number you are dividing by).*	 112 ÷ 8 = 80 ÷ 8 + 32 ÷ 8   1260 ÷ 6 = 1200 ÷ 6 + 60 ÷ 6  

Year 5 and 6 Division

Strategies & Guidance	CPA
Short division **Dividing a 4-digit numbers by 1-digit numbers** *The thought process of the traditional algorithm is as follows:* *How many 4s in 8? 2* *How many 4s in 5? 1 with 1 remaining so regroup.* *How many 4s in 12? 3* *How many 4s in 8? 2* *Warning: If you simply apply place value knowledge to each step, the thinking goes wrong if you have to regroup.* *How many 4s in 8000? 2000* *How many 4s in 500?* *100 with 1 remaining (illogical)* *The answer would be 125.* *Sharing the dividend builds conceptual understanding however doesn't scaffold the "thinking" of the algorithm.* *Using place value counters and finding groups of the divisor for each power of ten will build conceptual understanding of the short division algorithm.* *Area models are also useful representations, as seen with other strategies and exemplified for long division.*	8528 ÷ 4 2132 4 | 8 5 12 8 **Sharing**  8 thousands shared into 4 equal groups 5 hundreds shared into 4 equal groups Regroup 1 hundred for 10 tens 12 tens shared into 4 equal groups 8 ones shared into 4 equal groups. **Grouping**  How many groups of 4 thousands in 8 thousands? How many groups of 4 hundreds in 5 hundreds? Regroup 1 hundred for 10 tens. How many groups of 4 tens in 12 tens? How many groups of 4 ones in 8 ones?

Strategies & Guidance	CPA
Long division **Dividing a 4-digit number by a 2-digit number** *Follow the language structures of the short division strategy. Instead of recording the regrouped amounts as small digits the numbers are written out below. This can be easier to work with when dividing by larger numbers.* *If dividing by a number outside of their known facts, pupils should start by recording some multiples of that number to scaffold.*	34 12 | 408 36 48 48 0 408 ÷ 12